AD
TECHNICAL REPORT
TR 80-2

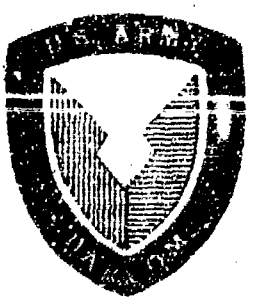
LEVEL II

10

AD A081

MATHEMATICS FOR SESAME MODEL

DTIC
ELECTE
MAR 14 1980
S D
C



U.S. ARMY
INVENTORY
RESEARCH
OFFICE

FEBRUARY 1980

ROOM 800
U.S. CUSTOM HOUSE
2nd and Chestnut Streets
Philadelphia Pa. 19106

DDC FILE COPY

Approved for Public Release; Distribution Unlimited

80 3 3 030

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): MATHEMATICS FOR SESAME MODEL	5. TYPE OF REPORT & PERIOD COVERED: Technical Report / 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Alan J. Kaplan	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: US Army Inventory Research Office, Room 800, US Custom House, 2nd & Chestnut Sts., Philadelphia, PA 19106	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 11
11. CONTROLLING OFFICE NAME AND ADDRESS: US Army Materiel Development & Readiness Command, 5001 Eisenhower Avenue, Alexandria, VA 22333	12. REPORT DATE: February 1980 / 13. NUMBER OF PAGES: 55
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 158	15. SECURITY CLASS. (of this report): UNCLASSIFIED / 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report): 14 IRR-TR-80-2

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Information and data contained in this document are based on input available at the time of preparation. Because the results may be subject to change, this document should not be construed to represent the official position of the US Army Materiel Development & Readiness Command unless so stated.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Multi-echelon Systems
Operational Readiness
Algorithms

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

SESAME is a model which determines component/part stockage quantities to achieve a given weapon system target operational availability at least cost. This report provides the mathematical basis of SESAME. Companion reports are a user's guide, and a report documenting the computer program.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

405572

AD A081931

TABLE OF CONTENTS

	Page
TABLE OF CONTENTS	1
CHAPTER I INTRODUCTION	
1.1 Use of the Model	3
1.2 Development of the Model	3
1.3 Purpose/Organization of Report	4
CHAPTER II ASSUMPTIONS	5
CHAPTER III MATHEMATICAL OVERVIEW	
3.1 Basic Approach	10
3.2 System Operational Availability	11
3.3 Backorder Penalty Costs	11
3.4 Determining Expected Backorders	12
3.5 Finding the Optimum Solution	13
CHAPTER IV CALCULATION OF OPERATIONAL AVAILABILITY	
4.1 Notation	15
4.2 Issues Involved	15
4.3 EMF vs MCTBF: A Digression	16
4.4 Calculation of OA, SA: Case A	17
4.5 Calculation of OA, SA: Case B	17
4.6 Comparison of Formulas	18
4.7 Choice of Formula	19
CHAPTER V CALCULATION OF EXPECTED BACKORDERS	
5.1 Alternative Approaches	20
5.2 Explanation of TWOPT	20
5.3 Evaluation of Alternative Methods	23
CHAPTER VI OPTIMIZATION	
6.1 Single Item Optimization	33
6.2 Constraints on Optimization	35
6.3 Multi-Item Optimization	35
CHAPTER VII EXTENTIONS	
7.1 Overview	38
7.2 Support Structure	38

Accession For
NTIS GRA&I
DDC TAB
Unannounced
Justification
By
Distribution/
Availability Codes
Dist | Avail and/or special
A

TABLE OF CONTENTS (CONT)

	Page
7.3 System Structure	38
7.4 Failure Process	39
7.5 Maintenance Policy	40
7.6 Resupply Considerations	42
7.7 Usage/Environment	43
7.8 Relationship to Other Problems	44
7.9 Impacts on Simple Simon/TWOPT	45
REFERENCES	46
NOTE 1 FAILURES DURING INSTALLATION	48
NOTE 2 SOLVING MULTIPLICATIVE OBJECTIVE FUNCTION	50
DISTRIBUTION	53

CHAPTER I

INTRODUCTION

1.1 Use of the Model

DARCOM's SESAME (Selective Stockage for Availability, Multi-Echelon) model is of the multi-item, multi-echelon type. It determines by means of mathematical optimizing techniques how many of each component to stock at each type stockage point in the supply system, taking into account the potential impact of each backordered component on system down time. SESAME will stock to achieve any given weapon system target availability at least cost. Its application is by weapon system, primarily to "ERPSL" weapon systems. These are systems for which stockage based on routine supply rules, geared to overall supply performance, is inadequate to permit achievement of desired weapon system operational availabilities. Typically, these systems have low densities, have reliable components, and are of great military importance.

The SESAME model is used in both budget and production modes. In budget mode its primary purpose is to develop curves showing the relationship between target operational availabilities and necessary inventory investment. In production mode it produces punch cards by which the stockage quantities it computes are entered into the Provisioning Master Record, a Commodity Command Standard System (CCSS) file. CSSS programs use this information in preparing Essential Repair Parts Stockage Lists (ERPSL's) for the field units, and in computing buy requirements for the wholesale system. The ERPSL's give stockage quantities by unit.

Part support is only one factor determining operational availability. The SESAME model has been incorporated into a life cycle cost model [16] so that all the relevant factors can be considered at one time, interactively. When SESAME is used in budget mode, the impact on operational availability of such factors as system reliability, and maintenance policies and times, has already been determined.

1.2 Development of the Model

For the most part the model is a synthesis of already existing mathematical techniques incorporated into a package designed to maximize user

convenience, and compatibility with CCSS. This package was designed by the DARCOM Provisioning Technical Working Group, which is a user group chaired by DARCOM HQ and incorporating various elements of organizational expertise; e.g., IRO has been the primary source of mathematical expertise and overall technical guidance. The model was actually programmed at IRO; US Army Armament Readiness Command made a number of program enhancements, and US Army Missile Command and Army Logistics Management Systems Agency developed programs to provide input data.

While the SESAME model is currently the most widely used model for Army ERPSL systems, it does not address various complicating factors which may be significant for particular weapon systems. This is by design, since it was felt that an all purpose model would be too complex, more difficult to use, and more difficult to maintain error free. Other comparable models are available [4, 9, 18], drawing on much of the same mathematical content, but with different strengths and weaknesses than SESAME. Guidelines are being developed for when each model is most appropriate, and when it may be necessary to tailor SESAME or one of the other models to a particular weapon system.

1.3 Purpose/Organization of Report

This report is intended to complement an extensive user guide [6] being written by a sub-committee of the DARCOM Provisioning Technical Working Group. This report discusses the mathematical content of SESAME. The chapters on assumptions and mathematical overview are intended for the general technical reader. The other chapters are intended for the inventory modeller.

There is still a third related report which provides actual computer program documentation for the SESAME program [7].

CHAPTER II

ASSUMPTIONS

Table 2.1 is a list of complicating factors which are relevant to the design of a multi-echelon model. Checked factors are incorporated into the current version of SESAME. Factors with a "$\sim$" were considered in model design and can be handled in an approximate fashion.

SESAME could be adapted to handle just about any of the other factors, in some cases exactly, in other cases approximately. Since each new factor complicates the model, this hasn't been considered desirable; further evolution of SESAME is expected. Chapter 7 discusses the difficulties of a mathematical nature, if any, presented by each of the untreated factors.

The rest of this chapter is a gloss on the less obvious complicating factors, and on the use of the "$\sim$."

Support Structure. By units differing, we refer to differences among units of the same echelon, e.g., in order and ship times. A non-vertical supply structure is one in which an echelon has a maintenance but not a resupply mission. It stocks to support its own mission. SESAME assumes echelons are not bypassed so times are additive: If ship time (given there are no backorders) from Depot to Direct Support Unit is 30 days, and from Direct Support Unit to Organization is 5 days, SESAME assumes that if nothing is stocked at DSU, then ship time from Depot to Organization is 35 days. If echelons can be bypassed, this time could be less.

System Structure. An application is a particular use of a component in a system. The pre-processor for SESAME [17] rolls up applications so that there is the same one line of information about each component. An indenture level refers to the hierarchical role of a component in a system. Typically, a second indenture component/part is used to fix a first indenture component which is used to fix the end item. SESAME computes stockage on lower indenture parts based on economic considerations, but does not explicitly model their contribution to system down time. Under consideration is a proposal to compute and output the expected average delay in getting these parts, so that it could be incorporated into the repair times of the reparable components. There would be one delay estimate

made for each echelon, being the average delay for all components/parts used at that echelon to repair other components.

Failure Process. Failures occur at random times. Some component failures may induce other components to fail so that failures are not independent and there are multiple removals; or, because of mechanic's error or limitations of diagnostic equipment, it may be likely that if component A fails, component B will also be removed and replaced. SESAME allows for differences between the number of system failures and total number of component failures/removals in its approach to computing operational availability. All component failure rates used in SESAME are actually removal rates.

Maintenance Policy. The same component may not always be removed, or may not always be repaired at the same echelon. Preventive maintenance is another reason there are removals without failures and is therefore treated by SESAME to the same extent that non-independent removals are. Under cannibalization, an inoperative system is used as a temporary source of components, if they are needed and otherwise unavailable.

Normally, it does not matter if the time to repair a system includes time to remove the failed component--it has no effect on any calculations. However, if this time is significant, and if diagnosis can be made before removal, an adjustment could be made to improve results (Chapter 7).

On many systems, test equipment narrows source of a problem to a set of components. These must be sequentially replaced until the problem is found.

Resupply Considerations. If an organization (ORG) requests a component from a direct support unit (DSU), and the DSU is temporarily out of stock, the ORG wait depends on what is due-in to the DSU. Some multi-echelon models assume the ORG request will be delayed the full order and ship time from the depot to the DSU. The issue of independence of successive resupply times is treated in detail in Chapter 5. Under lateral resupply, a unit's needs may be filled from stock held by other units at the same or lower echelons.

Under real time distribution, the supplier overrides first come, first served, depending on inventory positions of customers at the time of issue.

Usage/Environment. In general, SESAME is a steady state model, which does not accommodate changes in its inputs over time; it will compute the budgetary impact of phased deployment.

The standard definition of operational availability refers to per cent of calendar time the system is operational.

Relationship to Other Problems. Operational readiness float is a pool of systems held at DSU and issued when a system is returned to DSU for repair there.

TABLE 2.1: COMPLICATING FACTORS

(1) SUPPORT STRUCTURE
- ✓ (A) MULTI-ECHELON
- ✓ (B) EACH ORG (USING UNIT) SUPPORTS MORE THAN ONE OF THE WEAPON SYSTEM
- ✓ (C) UNITS DIFFER BY AREA
- (D) UNITS WITHIN THE SAME AREA DIFFER
- (E) NON-VERTICAL SUPPLY STRUCTURE
- (F) ECHELONS CAN BE BYPASSED TO SAVE TIME

(2) SYSTEM STRUCTURE
- ✓ (A) MULTIPLE COMPONENTS
- ∿ (B) COMPONENT WITH MORE THAN ONE APPLICATION
- ∿ (C) MULTIPLE INDENTURE LEVELS
- (D) REDUNDANCY

(3) FAILURE PROCESS
- ✓ (A) FAILURES ARE STOCHASTIC (RANDOM)
- ∿ (B) NOT ALL FAILURES/REMOVALS ARE INDEPENDENT
- (C) TIME BETWEEN FAILURES IS NOT EXPONENTIAL
- (D) MEAN FAILURE RATE IS NOT KNOWN, ONLY ESTIMATED

(4) MAINTENANCE POLICY
- ✓ (A) COMPONENTS ARE NOT ALWAYS REMOVED AT SAME ECHELON
- ✓ (B) COMPONENTS ARE NOT ALWAYS REPAIRED AT SAME ECHELON
- ∿ (C) THERE IS PREVENTIVE MAINTENANCE
- (D) CANNIBALIZATION IS UTILIZED
- (E) DIAGNOSIS CAN OCCUR BEFORE REMOVAL
- (F) FAILURE IS DIAGNOSED BY SEQUENTIAL REPLACEMENT

(5) RESUPPLY CONSIDERATIONS
- ✓ (A) TIME ON BACKORDER DEPENDS ON DUE-IN POSITION OF THE SUPPLIER
- ✓ (B) SUCCESSIVE RESUPPLY TIMES FOR A COMPONENT ARE NOT INDEPENDENT
- (C) USE OF EXPEDITED RESUPPLY
- (D) BATCH ORDERING OR REPAIR
- (E) LATERAL RESUPPLY
- (F) REAL TIME DISTRIBUTION

TABLE 2.1 (CONT)

(6) USAGE/ENVIRONMENT

✓ (A) SYSTEM OPERATIONAL AVAILABILITY MUST BE DETERMINED

∿ (B) DEPLOYMENT IS CHANGING

(C) USAGE RATES ARE CHANGING

(D) NON-STANDARD DEFINITIONS OF OPERATIONAL AVAILABILITY

(E) STATES OF PARTIAL DEGRADATION MUST BE MODELLED

(7) RELATIONSHIP TO OTHER PROBLEMS

(A) COMPONENTS ARE COMMON TO OTHER SYSTEMS

(B) MOBILITY OR OTHER CONSTRAINTS ON COMPONENTS STOCKED

(C) OPERATIONAL READINESS FLOAT IS UTILIZED

(D) REPAIR CAPACITY IS LIMITED

CHAPTER III

MATHEMATICAL OVERVIEW

3.1 Basic Approach

The objective of the model is:

$$\text{Minimize} \quad \sum_I \sum_J \text{Stock}(I,J) \times N(J) \times \text{Unit Price}(I)$$

$$\text{Subject to PNORS} \leq \alpha$$

where

Stock (I,J)	=	amount of item I stocked at an echelon J unit
N(J)	=	number of stocking units at echelon J
Unit Price (I)	=	unit price of item I
PNORS	=	% of time system is down due to unavailability of a component
α	=	maximum permissable PNORS

The PNORS constraint is included in the objective function by restating the problem as:

$$\text{Minimize} \quad \sum_I \sum_J \text{Stock}(I,J) \times N(J) \times \text{Unit Price}(I)$$

$$+ \sum_I \sum_J \text{Expected Backorders}(I,J) \times \text{RTD}(I,J) \times N(J) \times \text{Backorder Penalty Cost}(I)$$

where

Expected Backorder (I,J) = expected amount of item I backordered at echelon J

RTD(I,J) = replacement task distribution percent.

The replacement task distribution is a standard Army provisioning element indicating where the component is removed and replaced; e.g. RTD(I,1) = 100% means component is used entirely at echelon 1 (ORG).

There are now three questions:

(a) How is sytem operational availability calculated once the model is solved? Solving the model means finding the stockage quantities

for each component which minimize the sum of backorder and investment costs. Operational availability is the percent of calendar time the system is operational.

(b) How are backorder penalty costs determined and why does solving the restated problem really give a solution to the original problem?

(c) How is the optimum solution to the restated problem found? These questions are discussed non-mathematically in this chapter, and then in more detail in subsequent chapters.

3.2 System Operational Availability

From the expected backorders for a component, it is possible mathematically to determine the "average logistics down time" at user level for that component, i.e., the average time to get that component when it is needed. By weighting by the yearly removal rates of each component, it is possible to determine an overall average logistics down time. This is combined with average time between system failure and with down time while system is in repair to compute operational availability.

In these calculations only LRU's are considered. LRU was originally an Air Force term meaning line replaceable unit. To SESAME users it is an essential component whose removal and replacement restores the system to an operable condition, as opposed to a component used to fix a higher assembly which itself is a component. A carburetor might be an LRU while a float valve used to repair the carburetor is not an LRU.

In Chapter II, section on System Structure, treatment of non-LRU's was discussed. In that chapter they were less accurately referred to as lower indenture components. In some cases a second indenture component could be an LRU because it and not the first indenture item is always removed and replaced when it fails. The user guide provides additional information on LRU coding, and the designation can be made by an automated program.

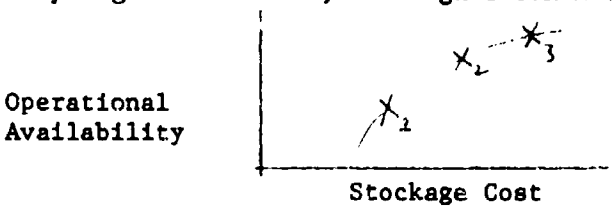
3.3 Backorder Penalty Costs

Backorder penalty depends on whether or not the item is an LRU. The backorder penalty for all LRU's is the same* since unavailability of any LRU

*See Chapter VI for a minor qualification.

has the same effect: it downs a system. The penalty cost is called the "CURPAR" for reasons to be explained.

When the restated problem is solved for a particular CURPAR there is, corresponding to the solution, a total inventory investment and total expected backorders. Since it is an optimum solution, it is known no other pattern of stocking costing less could result in fewer expected backorders. We have a least cost solution, but expected backorders may not correspond to our target operational availability. If the backorders are too high, we raise the backorder penalty (CURPAR) and get another solution. The new solution will spend more on stockage since each backorder avoided now reduces costs by a greater amount, the higher CURPAR. A curve is developed



X_1, X_2, X_3 correspond to solutions found with successively higher CURPAR's.

From the curve, the CURPAR needed to just achieve the Operational Availability is found. Since SESAME is so fast, little computer time is lost in generating a whole curve. The CURPAR selection process could easily have been automated, but the curve itself is of interest to the decision maker.

The backorder penalty for a non-LRU is equated to the cost of its next higher assembly. The rationale for this is that if the average number of backorders for a non-LRU is increased by one, we must invest in at least one additional next higher assembly to compensate for the additional assembly lying unused somewhere awaiting the non-LRU so it can be fixed.

3.4 Determining Expected Backorders

This is highly mathematical. Backorders depend on the demand rate at user level, stockage at user level, repair turn around time if the item is repairable at user level, and order and ship time to get the item from the user's supplier at the next echelon. The order and ship time depends not only on transportation times, but on whether the next echelon supplying unit is in stock. SESAME actually works from the top echelon downward. Suppose the support structure consists of ORG, DSU, wholesale. First wholesale

performance is determined. This permits us to calculate the order and ship times the DSU will experience. Then DSU supply performance is calculated so we can determine the order and ship times the ORG will experience and calculate user backorders.

3.5 Finding the Optimum Solution

The objective function - the expression for what is being minimized - is what a mathematician calls separable: for a given CURPAR, the solution for each component can be found separately from the solutions of every other component. This greatly simplifies things, yet tradeoff between components is still accomplished. We illustrate with two systems, each with two components, and identical input data except for the unit price of the second component (the second component is different on the two systems, but has the same failure rates, etc.).

	System A	System B
Component 1	$150	$150
Component 2	$100	$600

Suppose also we have that for either system:

	Alternative Stockage Quantities to Achieve Target Availability	
Component 1	5	8
Component 2	4	3

i.e. by stocking more of component 1 we can make do with less of component 2.

The optimum stockage for system A is (5,4)* at a cost of 5 x $150 + 4 x $100 = $1150. For system B, (5,4) would cost $3150 while (8,3) costs $3000, so (8,3) is optimum.

SESAME would reach the solution as follows:

	Stockage System A	Stockage System B
CURPAR - 1	3,2	3,1
CURPAR - 2	5,4	5,2
CURPAR - 3	-	8,3

*5 of component 1 and 4 of component 2.

For a given CURPAR, SESAME stocks less of component 2 on System B than on System A because, while all other inputs are the same, its unit price is higher. Therefore, CURPAR-2 is not high enough to produce a solution for system B which will achieve the target operational availability. Going to CURPAR-3, more of component 1 is now stocked as well as more of component 2.

For any given component, and given CURPAR, the least cost solution is found by trying all possible solutions which might be optimum and comparing costs. This is feasible because the computer can evaluate each solution very quickly, and it is possible to use mathematics to rule out many solutions as not being optimum without actually evaluating them.

CHAPTER IV

CALCULATION OF OPERATIONAL AVAILABILITY

4.1 Notation

OA: "Operational Availability"; hours system is in an up status as a per cent of total hours.

SA: "Supply Availability"; per cent of hours system is not down due to unavailability of a component.

IA: "Inherent Availability"; per cent of hours system is not down due to need to remove and replace an (available) component.

EMF: "Effective Maintenance Factor"; number of LRU removals per end item per year.

SFF: "System Failure Frequency"; number of times system goes down per year.

LDT: "Logistics Down Time"; average time to get an LRU when it is needed to repair a system.

MTTR: "Mean Time To Repair"; time to repair the system if all needed components are available.

MCTBF: "Mean Calendar Time Between Failures"; average time from when system goes up until next failure.

"i": Subscript "i" attached to any of the above denotes component i.

4.2 Issues Involved

There are two alternative formulas for computing operational availability. To rigorously derive either formula it must be assumed that failures are independent and there are no multiple removals of components (see Chapter 2). What we label the "Case A" formula also assumes that while the system is down because of component i, no other components can fail. The "Case B" formula assumes failures of other components are unaffected. In the real world, situations exist corresponding to both Case A and Case B and to mixtures.

In this chapter we present formulas for both cases, present evidence that choice of formula does not have much effect on the OA computed, and rationalize choice of the Case A formula as likely to be more robust.

But, first we provide some background information on the relationship between EMF and MCTBF.

For ease of exposition, we assume all system failures result in component removals. If they do not, imagine a component "x" with zero LDT and failure rate equal to the rate of system failures without removals.

4.3 EMF vs. MCTBF: A Digression

The following relationships hold for both Case A and Case B* for general continuous failure distributions.

$$(4.3.1) \qquad MCTBF = \frac{1}{\Sigma \frac{1}{MCTBF_i}}$$

$$(4.3.2) \qquad SFF = \frac{OA}{MCTBF} = OA \ \Sigma \frac{1}{MCTBF_i}$$

Equation (4.3.1) states that time between failures is the reciprocal of failure rate, and that system failure rate is the sum of component failure rates, i.e., the $(1/MCTBF_i)$. Equation (4.3.2) states that the number of system failures per year is the fraction of a year a system is up divided by the mean time between failures. It also implies (2nd equality) that the proportion of system failures caused by component i is the relative failure rate of component i, i.e., if component A's failure rate is twice component B's, it will cause twice as many system failures as component B. This is not obvious.

The difference between Case A and B is that in Case A all removals correspond to system failures while in Case B component failures and removals can occur when the system is already down (in fact, a percent equal to 100%-OA occur when the system is down).

$$(4.3.3a) \qquad EMF = SFF = \frac{OA}{MCTBF} \qquad \text{Case A}$$

$$(4.3.3b) \qquad SFF = (EMF)(OA)$$

or

$$EMF = \frac{SFF}{OA} = \frac{1}{MCTBF} \qquad \text{Case B}$$

*Relationships were derived by Barlow and Proschan [1] and Ross [21] as cited in Barlow and Hodes [2].

4.4 Calculation of OA, SA: Case A

Formulas used are:

(4.4.1) $$OA = (MCTBF)/(MCTBF + LDT + MTTR)$$

(4.4.2) $$SA = 1 - (LDT)/(MCTBF + LDT + MTTR)$$
$$\approx MCTBF/(MCTBF + LDT)$$

Equation (4.4.1) is well known (cf Barlow and Proschan). Equation (4.4.2) follows from the same arguments. The approximation is used to permit calculation of SA independently of MTTR. The ratio of the precise value to the approximate value is, after algebra:

(4.4.3) $$\text{Ratio} = 1 + \frac{(MTTR)(LDT)}{(MCTBF)(MCTBF + LDT + MTTR)}$$

which is close to 1 for high OA systems; e.g., if MTTR/MCTBF and LDT/MCTBF are each 5%, ratio is about 1.0025.

To use the formulas, the LDT_i are calculated (Chapter V). Then

(4.4.4) $$LDT = \Sigma(LDT_i/MCTBF_i) \div \Sigma\,(1/MCTBF_i)$$

i.e., the LDT_i are weighted by failure rates. This is justified by the observations made concerning the meaning of the second equality of (4.3.2).

4.5 Calculation of OA, SA: Case B

Formulas are:

(4.5.1) $$OA = \prod_i \frac{MCTBF_i}{MCTBF_i + LDT_i + MTTR_i}$$

(4.5.2) $$SA = \prod_i \frac{MCTBF_i}{MCTBF_i + LDT_i}$$

In other words, each component is up a given percent of time, its status (up or down) is independent of the status of all other components, so system availability is the product of the component availabilities. The formulas do assume a component does not fail during storage or installation. As a matter of theoretical interest, a correction to allow for failure during installation is derived in Note 1.

In the exact form, use of equation (4.5.1) would require input of the $MTTR_i$ to SESAME. An approximation eliminates this need:

$$(4.5.3) \qquad OA \approx (SA)(IA) = \prod_i \frac{MCTBF_i}{MCTBF_i + LDT_i} \prod_i \frac{MCTBF_i}{MCTBF_i + MTTR_i}$$

Interestingly the ratio of the precise value to the approximate is exactly the same as found in (4.4.3).

Equation (4.5.2) can be recast in a very different looking form as suggested by Bernard Price. It can be shown (cf Chapter V) that

$$(4.5.4) \qquad TWB_i/n = (LDT_i)(EMF_i)$$

where TWB_i is expected time weighted backorders for the ith component and n is the number of systems supported. Viewing item i as a system unto itself, so, for example, $SFF = EMF_i$, and referencing equation (4.3.2),

$$(4.5.5) \qquad EMF_i = \frac{(MCTBF_i)/(MCTBF_i + LDT_i + MTTR)}{MCTBF_i} = \frac{1}{MCTBF_i + LDT_i + MTTR_i}$$

Hence, combining (4.5.4) and (4.5.5).

$$(4.5.6) \qquad \frac{TWB_i}{n} = \frac{LDT_i}{MCTBF_i + LDT_i + MTTR_i} \approx \frac{LDT_i}{MCTBF_i + LDT_i}$$

and, referencing (4.5.2)

$$(4.5.7) \qquad SA \approx \prod_i (1 - TWB_i/n)$$

This form of the Case B approach is extensively used by Logistics Management Institute in its multi-echelon work.

4.6 Comparison of Formulas

Some sample SESAME runs were used to compare the Case A formula and Case B formula. These runs had only supply information, so the comparison was made between the SA formulas. MCTBF was determined by (4.3.3b), so what the comparison shows is how much difference use of (4.4.2) versus (4.5.2) makes when Case B assumptions are correct.

There was not much difference, as seen in the table below:

System	EMF	LDT	Case B Formula	Case A Formula
Generator	70.58	.464 Days	91.4%	91.8%
Missile	96.39	.141 Days	96.3%	96.4%
Tank	4.98	8.29 Days	89.3%	89.8%
Different Missile	26.91	.061 Days	99.55%	99.55%

4.7 Choice of Formula

The advantage of the Case A formula is that it permits use of an estimate of MCTBF which is not derived from or implicit in the $MCTBF_i$. For example, suppose a number of non-LRU components are incorrectly coded as LRU's. This would erroneously increase the estimate of EMF, but the estimate of MCTBF, which might originally be a system specification, and is then verified by testing, would still be correct. Using the Case A approach, the only error from a miscoded LRU is that the calculation of LDT would average in LDT_i not corresponding to LRU's, which might or might not affect the average. Under the Case B approach, availabilities of the non-LRU's would erroneously be factored in, lowering the overall estimate of OA.

If there are multiple removals, use of the Case A approach errs in that the expected time to get n components is larger than the time to get only one, yet Case A formula would input an LDT based on the latter. Under some circumstance this might not even be a source of error. Multiple removals may arise when components are replaced on a trial and error basis until the one faulty part is found. If an unavailable part is skipped in the search, it is only the LDT of the faulty component that is of concern. In any event, there are various ways to estimate the LDT to get n components (cf Chapter 7).

The Case B approach, under multiple removals, once again errs in implicitly assuming that each removal corresponds to a system failure with probability OA (cf 4.3.3b).

CHAPTER V

CALCULATION OF EXPECTED BACKORDERS

5.1 Alternative Approaches

Expected backorders are calculated for one component at a time. There are a number of procedures for doing this including METRIC [22], ACCLOGTROM [9], TWOPT and Simple Simon [14]. Simple Simon is most precise, while TWOPT performs best among the other three, and requires significantly less computer time and storage than Simple Simon. It is likely to be more adaptible than Simple Simon to inclusion of additional complicating factors. Both TWOPT and Simple Simon modules are available to users of SESAME.

The source of error in METRIC is its assumption that the order and ship times experienced are independent from requisition to requisition. Clearly, if you learned that a requisition was backordered 10 days, this conveys information about what delay a requisition placed, say one day later, will experience. The independence assumption causes METRIC to overestimate achievable item availability.

The source of error in ACCLOGTROM is that it does not account for the role of due-ins in reducing backorder times. This was discussed in Chapter II, section on Resupply Considerations.

Simple Simon is an exact model. However, it assumes Poisson demand from an infinite population, i.e., it does not account for a reduction in demand when a system goes down. Work done by this author, such as that reported in [10], indicates this is a minor concern. (See Appendix IV of reference, Table III, "Warm Standby")

Figures 1 thru 5 give Notation and then recursive formulas for calculating expected backorders by each of the approaches mentioned. Only Two Point will be discussed here, as the others are documented elsewhere. The ACCLOGTROM approach, as shown here, is actually a generalization of the approach in [9].

5.2 Explanation of TWOPT

Notation is in Figure 5.1, and the algebra by which expected backorders are computed is in Figure 5.2. To simplify the explanation, it is assumed

all removals occur at user level; the generalization of the algebra is straightforward. We will also assume the support structure consists of ORG and DSU echelons and a depot so NECH = 3.

Our objective is to calculate TWB_1. To do so we use equations 2a, 2b for J = 1. These equations require in addition to the basic data - DEN_1, EMF, P_1, TAT_2, $PSUM_1$, OST_1 - estimates of $CDEL_1$ and $FILL_2$ where $CDEL_1$ is the average delay when the DSU backorders an ORG demand, and $FILL_2$ is the DSU fill rate. To get these estimates we must use equations 2a thru 2d recursively beginning with the depot, J = 3. At the end of our depot calculations we have $FILL_3$ and DEL_2, from equations 2c and 2d respectively. We then recycl through 2a thru 2d for J = 2, getting $FILL_2$ and $CDEL_1$ as required. In doing our depot work, we need $FILL_4$ and $CDEL_3$, but these are given as initial conditions. They state that when the depot orders from the manufacturer there is a fixed lead time, with no chance of delay.

Let us now focus on 2a thru 2d. The due-in to a stock point, from repair and/or from the next echelon, is represented as a Poisson process compounded by a two point distribution on the mean. The two points result from a simplified representation of the actual continuous distribution on the OST experienced. U_{J1} corresponds to an OST equal to the input OST, which assumes no delay, and U_{J2} corresponds to the input OST augmented by the average time on backorder, given there is a backorder.

For those readers unfamiliar with Army provisioning terminology, we will illustrate the computation of U_{J1} for a DSU, J = 2. Then (DEN_2) (EMF) is the rate of removals for the component on all systems supported by the set of ORG's supported by a DSU. P_2 is the percent of these removals which result in unserviceables being returned and going into the DSU repair pipeline. TAT is the time from failure until the unserviceable has been repaired and returned to DSU stock. Hence $(DEN_2)(EMF)(P_2)(TAT_2)$ is the average amount due in from repair.

$(1\text{-}PSUM_J)$ is the percent of all removals not resulting in unservicables which can be repaired at either ORG or DSU, i.e., the percent of removals which represent draw-downs on DSU assets which must be made up by ordering from the depot. $(DEN_2)(EMF)(1\text{-}PSUM_2)(OST_2)$ is therefore expected due-in to the DSU from depot.

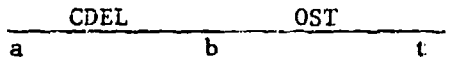
Equation (2b) is first justified for the case of $FILL_{J+1} = 100\%$. Time weighted backorders is the expected number of backorders at a random point in time. The probability of $(X-S_J)$ such backorders is the probability that total due-in is X: total assets, the sum of on-hand + due-in minus backorders is always S_J, so if due-in is X, either:

$$X \leq S_J \rightarrow \text{on hand is } S_J - X$$

$$X > S_J \rightarrow \text{backorders are } X - S_J$$

Many authors have shown that for the process under discussion the number due-in from repair is independent of the number due-in from procurement, or in this case the next echelon, and that the sum of due-in is Poisson distributed. All that must be assumed is that whether a given removal is repairable at echelon J is chosen by a Bernouilli trial with parameter P_J.

Equation (2b) is only approximately correct for $FILL_{J+1} \neq 100\%$. If we are at a random point in time t:



it assumes that either all requisitions initiated in the interval (a,b) were backordered, or none were backordered. In this sense it overcorrects for the METRIC assumptions of independence between requisitions. However, because it uses a two-point distribution to represent a continuous OST distribution, with tails, it still overestimates availability.

Derivation of (2c) is analogous to (2b), reflecting only the difference in computing fill rate and time weighted backorders.

Equation (2d) states that average backorder time per demand is (TWB)/(Demands Received) so that delay, given there is a delay, is (TWB/Demands Received)/$\overline{FILL}_J$. The first claim is the well known "λ = LW" relationship most simply and generally proved by Stidham [23]. The second claim follows from:

Ave Delay = (Delay Given There is a Delay)(Probability of Delay)

Note that since LDT is average delay, we must multiply by $\overline{FILL}_1$ in (3b) to get average delay back from $CDEL_0$.

5.3 Evaluation of Alternative Methods

At the time of the evaluation, we had a somewhat limited data base consisting of 301 Patriot LRU's. In order to extend the generality of the analysis, this data was also made to "resemble" a Howitzer by changing system parameters. Details are in Table 5.1.

To derive Table 5.2 each of the approximate methods were run for a range of CURPAR's. For each solution they obtained, the stockage determined was also evaluated using Simple Simon.* Reported in Table 5.2 are the weapon system supply availabilities projected by the approximate methods, and the exact supply availabilities computed by Simple Simon for the same stockage. In the Patriot "MF x 4" run, all removals rates were multiplied by 4. Analysis of the earlier results suggested this might constitute a worst case.

Table 5.3 shows actual cost comparisons of inventory investment needed to achieve various levels of weapon system supply availability when stockage is determined using each of the methods. For example, on the Patriot, it would cost $26.5 million to achieve 90% weapon system supply availability using SESAME with the METRIC backorder evaluation modules as one of its subroutines. Note that METRIC would project higher than 90% for $26.5 million. After SESAME with METRIC was run, Simple Simon was used to calculate the solution and compute true availability.

TWOPT performed very well, with little loss in accuracy or increase in cost over Simple Simon. Use of a constraint on stockage may have contributed to this. In Chapter VI two constraints on optimization used in SESAME are explained. In this research the SIP lower bound constraint was used, the other was not.

---

*W. Karl Kruse designed and programmed the evaluator, and did the Simple Simon runs reported in Table 5.3 as well as the programming for those runs.

All variables listed, except OST_J and DEN_J, vary by component.

J - echelon; J = 1 is user, etc.

S_J - stockage (on hand + on order) for unit at echelon J

U_J - mean pipeline qty (amount due-in) for unit at echelon J

OST_J - order and ship time to echelon J when echelon J + 1 is in stock

DEL_J - average increase in OST because of backordering at echelon J + 1

$CDEL_J$ - conditional DEL_J. Delay given requisition is backordered.

TAT_J - repair turn around time; includes time to evacuate item, if necessary, from point of removal to echelon J as well as time to repair it.

P_J - Maint Task Distribution fraction; percent of total removals which are repaired at echelon J.

$PSUM_J$ - $\sum_{j=1}^{J} P_j$

DEN_J - density supported by each unit at echelon J

EMF - effective maintenance factor (demand per day per end item)

D_J - demand on unit at echelon J

TWB_J - time weighted backorders = expected number of backorders at a random point in time

$FILL_J$ - Fill rate provided by unit of echelon J

$\overline{FILL}_J$ - $1 - FILL_J$

$p(x;u)$ - Poisson density for parameter u;

NECH - number of stocking echelons

LDT - average logistic downtime for component

FIGURE 5.1: NOTATION

Initial Conditions:

1a $DEL_{NECH} = 0$

1b $FILL_{NECH+1} = 100\%$

Recursive Formulas

2a $U_{J1} = (DEN_J)(EMF)\left\{P_J TAT_J + (1-PSUM_J)(OST_J)\right\}$

$U_{J2} = (DEN_J)(EMF)\left\{P_J TAT_J + (1-PSUM_j(OST_J + CDEL_J)\right\}$

2b $TWB_J = [FILL_{J+1}][\sum_{x=S_j}^{\infty} p(x;U_{j1})(x-S_J)] + [\overline{FILL}_{J+1}][\sum_{x=S_j}^{\infty} p(x;U_{J2})(x-S_j)]$

2c $FILL_J = [FILL_{J+1}][Pr(x < S_J;U_{J1}) + [\overline{FILL}_{J+1}][Pr(x < S_J;U_{J2})]$

2d $CDEL_{J-1} = \frac{TWB_J}{D_J} / \overline{FILL}_J$

Evaluation

3a $BACKORDERS = TWB_1$

3b $LDT = CDEL_o * \overline{FILL}_1 = TWB_1/D_1$

FIGURE 5.2: TWOPT

Initial Conditions

(1a) $FILL_{NECH+1} = 100\% \; ; \; Pr[BO_{NECH+1} > 0] = 0$

Recursive Formulas

(2a) $$Pr[BO_J = k] = Pr[X_J + Y_J = S_J + k] = \sum_{i=0}^{S_j+k} Pr[X_J = i] \; Pr[Y_J = S_J+k-i]$$

(2b) $$U_J = (DEN_J)(EMF)\,[P_J TAT_J + (1-PSUM_J)(OST_J)]$$

(2c) $$Pr[X_J = m] = p(m;U_J)$$

(2d) $$Pr[Y_J = m] = \sum_{n=m}^{\infty} \binom{n}{m} r_J^m \, (1-r_J)^{m-n} \, Pr[BO_{J+1} = n]$$

(2e) $$r_J = \frac{\text{Number of locations in Echelon J+1}}{\text{Number of locations in Echelon J}}$$

Evaluation

(3a) $$BACKORDERS = TWB_1 = \sum_{k=o}^{\infty} (k) Pr[BO_1 = k]$$

(3b) $$LDT = TWB_1/D_1$$

FIGURE 5.3: SIMPLE SIMON

Initial Conditions

(1) $DEL_{NECH} = 0$

Recursive Formulas

(2a) $$U_J = (DEN_J)(EMF)\left\{(P_J)(TAT_J) + (1-PSUM_J)(OST_J + DEL_J)\right\}$$

(2b) $$TWB_J = \sum_{x=S_J}^{\infty} p(x_j U_j)(x-S_j)$$

(2c) $$DEL_{J-1} = \frac{TWB_J}{D_J}$$

Evaluation

(3a) $Backorders = TWB_1$

(3b) $$LDT = \frac{TWB}{D_1}$$

FIGURE 5.4: METRIC

Initial Conditions

(1) $DEL_{NECH} = 0$

Recursive Formulas

(2a) $U_J = (DEN_J)(EMF)\left\{(P_J)(TAT_J) + (1\text{-}PSUM_J)\ (OST_J + DEL_J)\right\}$

(2b) $FILL_J = Pr\ (x < S_J;\ U_J)$

(2c)* $DEL_{J-1} = \overline{FILL_J}\ \dfrac{(P_J)(TAT_J) + (1\text{-}PSUM_J)(OST_J + DEL_J)}{1\text{-}PSUM_{J-1}}$

Evaluation

(3a) $BACKORDERS = (D_1)(DEL_0)$

(3b) $LDT = DEL_0$

*Note that $PSUM_0 = 0$ and that $P_J/(1\text{-}PSUM_{J-1}) + (1\text{-}PSUM_J)/(1\text{-}PSUM_{J-1})$ are weights summing to 1.

FIGURE 5.5: ACCLOGTROM

TABLE 5.1

EVALUATION DATA BASE

DATA BASE SOURCE

301 LRU's

PATRIOT REPARABLE ITEMS (ABOUT 1/2 OF ALL REPARABLE LRU's)

3 WITH MF*	500
13 WITH MF	100
138 WITH MF	10
163 WITH MF	10

	PATRIOT	"HOWITZER"
ECHELONS	3	3
MTD	0%, 0%, 97%	25%, 25%, 45%
WASHOUT	3%	5%
CST	2, 40, 570	10, 60, 570
TAT	-, -, 180	-, 7, 90
NO. UNITS PER ECH	96, 16, 1	210, 210, 1
DENS PER ORG	1	18
MF ADJUSTMENT		CUT MF IN HALF TRUNCATE MF AT 40

*MF IS MAINTENANCE FACTOR OR REMOVALS PER 100 END ITEMS PER YEAR.

TABLE 5.2

PROJECTION ERROR

PROJECTED ("EXACT")

	CURPAR	METRIC	ACCLOG	TWOPT
PATRIOT				
	$1.0 MIL	90.3 (88.7)	90.2 (91.2)	90.0 (89.6)
	$2.5 MIL	96.6 (95.5)	96.8 (97.1)	96.5 (96.2)
	$5.0 MIL	98.9 (98.1)	99.1 (99.1)	98.9 (98.7)
	$10.0 MIL	99.6 (99.0)	99.6 (99.6)	99.5 (99.4)
HOWITZER				
	$50,000	87.4 (86.4)	90.5 (90.8)	87.2 (87.0)
	$75,000	91.9 (90.8)	94.0 (94.2)	91.5 (91.2)
	$100,000	94.3 (93.3)	96.8 (96.9)	94.1 (93.9)
	$500,000	98.9 (98.5)	99.4 (99.4)	98.8 (98.7)

TABLE 5.2 (CONT)

PROJECTION ERROR

PROJECTED ("EXACT")

PATRIOT: MF X 4

CURPAR	METRIC	ACCLOG	TWOPT
$1.0 MIL	92.2 (86.7)	93.3 (94.3)	91.6 (90.3)
$2.5 MIL	97.8 (94.3)	98.0 (98.3)	97.5 (96.7)
$5.0 MIL	99.0 (96.4)	99.2 (99.4)	98.9 (98.4)
$10.0 MIL	99.5 (97.9)	99.7 (99.7)	99.5 (99.2)

TABLE 5.3

COST COMPARISON

PATRIOT

TARGET	METRIC	ACCLOG	TWOPT	SIMON
90%	26.5 MIL	27.0 MIL	25.9 MIL	25.8
95%	34.3	34.6	33.6 MIL	33.4
98%	43.0	42.5	41.4	40.7
RUN COST (4 CURPAR'S)	$12	$16	$15	$83

HOWITZER

90%	130 MIL	134 MIL	129 MIL	129 MIL
94%	150 MIL	156 MIL	149 MIL	149 MIL
99%	195 MIL	204 MIL	193 MIL	193 MIL
RUN COST	$26	$41	$37	$236

PATRIOT: MF x 4

90%	62.4 MIL	64.2 MIL	61.3 MIL	61.0
95%	69.0 MIL	69.8 MIL	67.2 MIL	66.5
99%	80.0 MIL	78.6 MIL	76.0 MIL	75.0
RUN COST	$25	?	$32	

CHAPTER VI

OPTIMIZATION

6.1 Single Item Optimization

The procedure discussed here was developed by Kruse [13] for a two echelon example and was formally generalized to more than two echelons by Kotkin [12].

The objective is to determine upper bounds on optimum stockage quantities, and then to dynamically lower these bounds as potentially optimum solutions are evaluated.

Notation

S_J = stockage at echelon J

$TC^*_{J-1}(S)$ = lowest possible sum of backorder and inventory costs, given $S_J = S$, where inventory cost is charged only for stock at echelons 1 thru (J-1).

UP = unit price

S^*_J = optimum stockage at echelon J

Procedure

Procedure is illustrated for n^{th} echelon. First calculate $TC^*_{n-1}(0)$ and $TC^*_{n-1}(\infty)$, where the latter is the cost when an echelon n stocker is always in stock, and the tormer is the cost when it is never in stock. Then

$$(6.1.1) \qquad (S^*_n)(UP) + TC^*_{n-1}(S^*_n) \leq (0)(UP) + TC_{n-1}(0)$$

since the left hand side is the total cost including stockage cost at echelon n if $S_n = S^*_n$, and the right hand side is the total cost if $S_n = 0$.

Hence, using (6.1.1)

$$(6.1.2) \qquad (S^*_n)(UP) + TC_{n-1}(\infty) \leq TC_{n-1}(0)$$

since $TC_{n-1}(\infty) \leq TC_{n-1}(S^*_n)$; i.e., as upper echelon stock is raised, delays

to lower echelons drop and therefore so do lower echelon costs.

By algebra from (6.1.2)

$$(6.1.3) \qquad S_n^* \leq \frac{TC_{n-1}(0) - TC_{n-1}(\infty)}{UP}$$

This constitutes the first upper bound on S_n^*. Suppose in the course of evaluating $S_1, S_2, \ldots$ a value S_n' is found such that

$$(6.1.4) \qquad TC_{n-1}(S_n') + (UP)(S_n') \leq TC_{n-1}(0)$$

We know

$$(6.1.5) \qquad TC_{n-1}(\infty) + (UP)\ (S_n^*) \leq TC_{n-1}(S_n') + (UP)(S_n')$$

or by algebra

$$(6.1.6) \qquad S_n^* \leq \frac{TC_{n-1}(S_n') + (UP)(S_n') - TC_{n-1}(\infty)}{UP}$$

which by (6.1.4) provides a lower bound on S_n^* than does (6.1.3).

Now if n is the upper echelon, we try all values for S_n until we reach an upper bound on S_n^*, keeping track of the lowest cost S_n found in the search. For each value of S_n we find $TC_{n-1}^*(S_n)$ by solving an n-1 echelon problem in which ship times are a function of S_n. For the n-1 echelon problem we use the bounding procedure all over again, so that a problem with any number of echelons can be solved recursively.

At the lowest echelon cost is a convex function of S_1 so a bounding procedure is not required. Also, for fixed values of S_J, $J \geq 3$, Kruse [13] showed S_1^* is a (weakly) monotonic decreasing function of S_2. This fact is used to save computer time.

Implication of Users Above ORG. A percent of LRU removals may occur above ORG, reflecting weapon system evacuations (refer to use of RTD(I,J) in restated problem in Chapter III). The bounding procedure must still work if the definition of $TC_{J-1}^*(S)$ is changed to include backorder costs incurred at n^{th} echelon if S_n is set to S. Backorder costs are evaluated as

$(TWB_J)(PCER)(CURPAR)$ where

TWB_J are expected backorders at n^{th} echelon

PCER is percent of demands on J^{th} echelon accounted for by removals there rather than resupply of lower echelons.

6.2 Constraints on Optimization

If a field unit experiences a frequency of demand for an item which equals or exceeds its stockage criteria, it will stock that item in an amount at least equal to what it would compute using routine replenishment rules regardless of what SESAME computes. By DoDI 4140.42, this entire amount cannot be provisioned, but a somewhat lesser amount determined by the SIP (Standard Initial Provisioning) model can be. For items which will pass stockage criteria, SIP quantities are used as lower bounds, to incorporate the realities of field stockage into the SESAME model. For most ERPSL systems this can have little impact since few items do pass stockage criteria, at least at lower echelons, and SIP stockage is limited. SESAME also has an option to use a lower bound which approximates post-provisioning field unit stockage computations.

Currently, SESAME is not used to compute wholesale quantities. Instead retail stockage is based on initial fill rates and delays actually experienced at wholesale level. There are a number of reasons for this but this policy is under review.

6.3 Multi-Item Optimization

Case A Approach

Using the Case A approach to Weapon System availability, our objective is to minimize inventory investment subject to a bound on LDT. We show this is equivalent to constraining on the sum of time weighted backorders, the problems we actually solve (cf Chapter III).

Now, recalling equation (4.4.4) and the notation of that Chapter

$$(6.3.1) \qquad LDT = \Sigma(LDT_i/MCTBF_i)/\ \Sigma(1/MCTBF_i)$$

Also, by (4.3.3a)

$$(6.3.2) \qquad EMF = \frac{OA}{MCTBF}$$

and because relative removal rates are proportional to relative failure rates, (cf discussion of Equation 4.3.2)

$$(6.3.3) \qquad EMF_i = \frac{(EMF)(1/MCTBF_i)}{\Sigma\ 1/MCTBF_i}$$

Substituting (6.3.3) into (6.3.1)

$$(6.3.4) \qquad LDT = \Sigma(LDT_i)\frac{(EMF_i)}{EMF}$$

Recalling Chapter IV,

$$(6.3.5) \qquad LDT_i = TWB_i/\text{Demands Received}) = TWB_i/(EMF_i)(N)$$

where N is number of weapon systems supported. Hence, by (6.3.4) and (6.3.5)

$$(6.3.6) \qquad LDT = \Sigma\ TWB_i/(EMF)(N)$$

Case B Approach

Using the Case B Approach to operational availability we would wish to minimize (recall Equation 4.5.7)

$$(6.3.7) \qquad SA = \prod_i (1 - \frac{TWB_i}{N})$$

It is of interest how a solution obtained to minimize (6.3.7) would compare with one to minimize LDT.

In Appendix 2 we show that the solutions must be close, and in fact, if in solving for item i CURPAR were adjusted by adding UP_i, so that backorders were costed out at (CURPAR + UP_i), we would be over correcting for the multiplicative objective function.

This adjustment was in fact made in SESAME, at a time when the Case B approach was the preferred approach, and has been retained. It has real impact only when component availability targets are not high, and helps moderate extreme differences between protection afforded high cost and low cost LRU's in that situation. This seems prudent as the Case B approach might be more correct for a given system, and if there is any cannibalization, extreme differences in targets is not desirable.

CHAPTER VII

EXTENSIONS

7.1 Overview

The purpose of this chapter is to briefly assess the mathematical difficulties presented by those complicating factors which SESAME does not treat. The reader is referred back to Chapter II and its discussion of what the untreated complicating factors are.

As in any such chapter, the reader proceeds at his own risk, since unproved and untested materiel is being presented. Furthermore, the whole may be greater than the sum of the parts; difficulties posed by treating several factors may be greater than implied by examining each factor by itself.

7.2 Support Structure

Difficulties presented by untreated factors are not of a theoretical nature.

7.3 System Structure

Redundancy of Like Items. Underlying SESAME's current objective function is the assumption that each backorder corresponds to another down system. Hence if p_j is the probability of j backorders (for part i), and each system down is evaluated at a cost of CURPAR,

$$\text{Cost} = (\text{CURPAR}) \left[\sum_j (j)(p_j)\right] = (\text{CURPAR})[\text{Ex}(j)]$$

With redundancy, depending on the redundancy structure and the number of system supported, there is some non-linear function f(j) which maps j into the expected number of systems down given j. Calculation of p_j itself is affected by redundancy.

Kaplan [10] discusses the computation of $p_j f(j)$ under redundancy. This expression would replace Ex(j) in the SESAME model, but can be much more difficult to compute and requires knowledge of MTTR_i (time to remove and replace component on system)

Redundancy of Unlike Items. In this case the system can operate, for example, if either component i or component j is up. For ease of exposition

assume one system supported per ORG. Using the Case B view of the world (Chapter IV), which is most appropriate in this context, and defining A_i and A_j to be the availabilities of components i and j, respectively, system availability is $A_i + A_j - A_iA_j$ rather than A_iA_j. This destroys the separability of the objective function (Chapter III) which underlies the SESAME approach to multi-item optimization.

Presumably, redundancy of unlike items would involve (possibly many) small subsets of items, i.e. only a few components could interchangeably do any one function. SESAME could approach optimization among these subsets by iteration. Continuing our example of two components, total cost for the subset would be evaluated as (Inventory Investment) + (CURPAR)$(A_i + A_j - A_iA_j)$ where the A_i, A_j depend on the p_i, p_j, the distributions on time weighted backorders. Once A_i had been determined, denote this A_i^1, the objective for component j is to minimize (Inventory Investment j) + (CURPAR)$(1-A_i^1)A_j$. Once the A_j resulting from this minimization is found, component i would be reevaluated and A_i^2 determined. More generally the objective is to minimize (Inventory Investment) + (CURPAR) $(p_1, p_2 \ldots p_n)$. The iteration procedure would stop when improvement in total cost dropped below a threshhold. No attempt has been made to show iteration would converge to the optimum solutions.

7.4 Failure Process

Time Between Failures is Not Exponential. The simplest generalization is to the case of independent, identically distributed interarrival times, so that demand constitutes a renewal process. If N(t) is the number of renewals during time t, the distribution for N(t) would be substituted for the cumulative Poisson in computing backorders. The expression for N(t) required is that for the equilibrium renewal process.

Dependent Removals. Dependent removals might result in demands for more than one unit at a time. This has been discussed in the literature with justification provided for use of Compound Poisson distributions to replace the Poisson in the evaluation of backorders, within the context of the METRIC (Chapter V) algorithm [22]. Unfortunately, we require not the average time to get a unit of stock, but the average time to get all units demanded in

one requisition. This is discussed by Kaplan [11a] for single echelon work, and is readily generalized via TWOPT.

Chapter IV discusses the error in using average logistics down time when more than one component type is required. This differs from requisition size greater than one since the times to get each component required are essentially independent. Computation of the time to get n components is relatively simple for reasonable approximations of the lead time process, e.g. a process consisting of a probability of zero time, and a uniform or exponential distribution on time if it is not zero.

Mean Failure Rate is Not Known, Only Estimated. Suppose there were two possible values for the failure rate of a component, with known probabilities. This can easily be incorporated into the optimization by evaluating all stockage levels given both possible demand rates and weighting backorders computed by the probability of each rate. This is not equivalent to first computing one expected demand rate and including the variability due to unknown mean in total variability about the estimated mean. More generally, the Compound Poisson distribution cannot be used to model unknown mean at each echelon as is sometimes claimed in the literature for this would not capture the correlation between mean demand at each echelon. Since SESAME is very fast, an entire histogram of possible true means can be accommodated.

An experiment was run on data for a Missile System in which stockage levels were computed assuming that the failure rate was known, but evaluated assuming it could be 25% higher or 25% lower with equal probability. This reduced projected system supply availability from 89.5% to 86.4% and from 99.4% (with a higher CURPAR) to 98.6%. Moreover, three high failure items had to be excluded. On these items the $\pm$ 25% experiment reduced availability precipitously; for the lower CURPAR the product of availabilities for the three items dropped from 99.7% to 91.4%.

7.5 Maintenance Policy

Cannibalization. Under complete cannibalization, to have at least n systems down because of supply unavailability there would need to be at least n backorders for one of the components. Without cannibalization, a total of n backorders on n different components could produce the same effect. In SESAME there would be no particular difficulty in computing the entire

backorder distribution for each component. Let

$F_i(X)$ = probability of no more than X backorders of component i

$$F(X) = \prod_i F_i(X)$$

Then, using a Case B type approach to supply availability, and a well known result of probability theory regarding expectations,

$$\text{Prob (No. Systems Down} \geq X) = 1 - F(X-1)$$

$$\text{Ex(No. Systems Down)} = \sum_{x=1}^{m} 1 - F(X-1), \quad m = \text{No. of Systems}$$

$$\text{Supply Availability} = 1 - (\text{Ex No. Systems Down})/m$$

The approach stated does not consider possibly delays due to removal of the component being cannibalized. It is possible to model this delay accurately if it is assumed:

a. Component is only cannibalized when needed to fix a system.

b. If a component arrives from supply it is installed on a deadlined equipment, even if it is not sufficient by itself to make the system operable.

c. Once cannibalization is begun, system down time equal to the removal time is incurred.

The first assumption is realistic, the second may be, while the third is not, in that a component might arrive from resupply before removal was completed. Use of the assumption overstates down time.*

Given the assumptions, the proportion of the year a system is down due to removal of component i may be calculated, and from this operational availability. Let OA, and IA be defined as in Chapter IV. Also, let $p_i(x)$ be the probability stock on hand is $= x$, and let R_i be removal time. Delay due to removal is incurred whenever there is a failure of component i, and stock on hand for i is ≤ 0 (negative stock denotes backorders), and there is a system from which the component can be cannibalized. If DR_i is yearly removal time per end item and there are EMF_i yearly component failures,

$$DR_i = (EMF_i)(R_i) \sum p_i(x)[1 - \prod_{j \neq i} F_i(x)]$$

*There is also an assumption made, of little consequence, that if a system fails you first try to cannibalize to fix it, rather than cannibalize it to fix another system.

The last term in brackets is the probability there is at least one component with at least X+1 backorders so that there are (X+1) systems down and component i, with X backorders, can be cannibalized if a demand occurs.

Then

$$OA \quad (IA)\left(1 - \frac{\text{Ex No. Systems Down}}{n}\right) \prod_i (1-DR_i)$$

The separability property of the objective function is destroyed by cannibalization, making optimization difficult. One approach would be to experiment with various forms of non-linear backorder functions, e.g. set cost = $(CURPAR)(TWB_i)^n$, $n > 1$.

Diagnosis Precedes Removal. Removal time must be considered part of "Mean Time to Repair" in calculation of operational availability. Still, if a part of MTTR precedes logistics down time, and a part follows it, this does not impact on the rationale of the operational availability formulas. However, if there is built-in diagnostic equipment so diagnosis precedes removal, then in some situations logistics down time and time to repair can proceed concurrently.

If R_i is removal time and LDT_i is logistics down time, both treated here as random variables, not expectations, then adjusted LDT_i is max $(0, LDT_i - R_i)$. To find the expectations of this variable requires the distribution on adjusted LDT_i (cf discussion on Dependent Removals).

7.6 Resupply Considerations

Expedited Supply. Suppose that d days between DSU and ORG can be saved by expedited resupply. This information might be used in a number of ways to adjust logistics down time downward.

a. If there is no ORG stockage, subtract d days.

b. As adjustment a, but also subtract (d)(1-ORG-FILL) (1-DSU-FILL) if ORG does stock. "FILL" refers to fill rate. The rationale is that if the ORG is out of stock, and there is not due-in from the DSU already enroute, i.e. the DSU is out of stock, then d days will be saved.

c. As adjustment b, but also subtract (1-ORG-FILL)(DSU-FILL)(f(d,t)) where t is routine resupply time between DSU and ORG and f is some function.

The assumption is that even if stock is enroute to the ORG, if it will not routinely arrive within d days, expediting can help. To determine f, it might be assumed time to receipt of the due-in is uniformly distributed between 0 and OST, or, considering the possibility of multiple due-ins, a Beta distribution might be used (if there are n Poisson arrivals in an interval, time to i^{th} is Beta distributed).

It is possible to incorporate expedited supply directly into the optimization routine.

Batch Ordering. A straight forward approach would be to compute the economic order quantity first, and then optimize on the total stockage objective, i.e., reorder point plus order quantity. At upper echelons the demand process would be Compound Poisson, to account for demands greater than 1 unit. Problems are:

a. Impact on validity of Two-Pt approximation approach. Simple Simon could handle exactly a two-echelon problem with only upper echelon using EOQ.

b. Demands on upper echelon are correlated if lower echelon follow batch ordering policies.

Muckstadt [19] has developed an exact solution which is computionally tedious and has never been implemented. For larger number of customers he suggests ignoring correlations. Durnmeyer and Schwarz [8] propose a heuristic which they tested by simulation.

Lateral Resupply. Problems could be horrendous.

7.7 Usage/Environment

Barzily and Gross [3] have done a study on coping with changing environments in a single echelon problem with limited repair capacity. Bein [4] has considered degradation. While different definitions of operational availability must be treated on a case by case basis, there is an entire class of weapon systems which are sortie oriented, so that the availability measure of interest is availability during sortie time. At one extreme such problems may degenerate to single echelon "fly-away" kit problems in which resupply is not much of a consideration. This occurs if time between sorties is large so

that resupply can always occur, while sortie time itself is small. At the other extreme this procedure might be used:

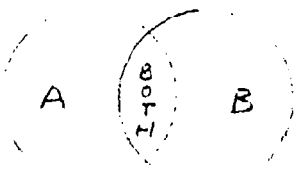
a. Compute logistics down time as currently.

b. Based on an approximate distribution on down time, on sortie time, and on time between sorties, compute the percent of down time which occurs during and not between sorties.

This approach ignores the impact of sorties on the failure process.

7.8 Relationship to Other Problems

Common Items. Suppose there were two systems with LRU's distributed:



One approach would optimize all three sets individually, over ranges of curve parameters, and then search for optimum combinations of curve parameters. Note that in computing expected logistic down time, the contribution of "common" would vary between weapon system A and B, depending on the non-common LRU's as well as the failure frequency on A and on B of each common LRU.

As the number of systems grows, the number of sets to be defined grows geometrically, e.g. on three systems there are parts common to A and B, B and C, A and B and C. In principle it should be feasible to cope with a great number of sets, but a simpler approach might be desirable. Such an approach is to first optimize for A, but also compute the contribution to down time on other systems of the LRU's on A common to them. Next B is optimized, and so on. In doing the optimization on A, all backorders are costed out at the CURPAR being used for A, even though some of the backorders impact on other systems.

Mobility or Other Constraints. Suppose there were a limit on the number of components to be stocked at a unit. Using the Lagrangian approach, a cost would be included in the optimization for each component stocked, and this cost would be raised to achieve desired stockage list size. Since now there

are two Lagrangians, pertaining to list size and backorder cost (CURPAR), the search process for correct values would probably need to be automated.

Operational Readiness Float. This is treated by Kaplan [11]. It is not particularly difficult.

7.9 Impacts on Simple Simon/TWOPT

The discussion of batch ordering raised the issue of how the factor impacted on Simple Simon or the TWOPT approximation. Simple Simon assumes Poisson demand, so a number of the other factors also bears on this. Robustness of TWOPT has never been tested.

REFERENCES

1 Barlow, Richard E. and Frank Proschan, "Availability Theory for Multi-Component Systems," in Multivariate Analysis - III, P. R. Krishnaiah (ed), Academic Press, New York, 1973.

2 ____ and Esther S. Hudes, "Asympototic Measures of System Performance Under Alternative Operating Rules, Vol I," University of California, Berkeley, 1975.

3 Barzily, Zeev and Donald Gross, "Transient Solutions for Repairable Item Provisioning," George Washington University, Washington, DC, 1979.

4 Bein, Larry, "Firefinder Model," Stewart Associates, Boston, 1979.

5 Clark, Andy J., "Logistics Support Economic Evaluation: Optimal Operational Availability Inventory Model," CACI, Inc., Arlington, VA, 1978.

6 Cohen, Martin with Robert Jones, Jerry Peek, "SESAME User's Guide," DARCOM, 1980. (to be published)

7 ____, "SESAME Program Documentation," Army Inventory Research Office, Philadelphia, PA, 1980 (to be published).

8 Deuermeyer, B. L. and Leroy B. Schwartz, "An Analysis of System Service Level in Warehouse-Retailer Distribution Systems: The Identical Retailer Case," Paper presented to Multi-level Production/Inventory Systems Conference, Purdue University, 1979.

9 Forrey, Kingsley E., "An Analytic Model for Determining Optimum Site Stockage Levels," presented at Joint ational Meeting of TIMS/ORSA, Spring, 1979.

10 Kaplan, Alan J., "System Availability With Redundancy and Spares," Army Inventory Research Office, Philadelphia, 1979.

11 ____, "ORF vs ERPSL Tradeoffs," Army Inventory Research Office, Philadelphia, 1980. (to be published)

11a ____, "Commuting Expected Values of Customer Requests Backordered," Management Science, 1971, p647-651.

12 Kotkin, Meyer, "On the Optimal Stock Levels in a Multi-Echelon Maintenance System," Army Inventory Research Office, Philadelphia, 1978.

13 Kruse, W. Karl, "An Algorithm for Computing Optimum Stock Levels in a Two Level Maintenance System," Army Inventory Research Office, Philadelphia, 1970.

14 Kruse, W. Karl, "An Exact N Echelon Inventory Model: The Simple Simon Method," Army Inventory Research Office, Philadelphia, March 1979.

15 Logistics Management Institute, "Measures of Military Essentiality," Washington, DC, 1972.

16 Metzger, James, "US Army Model Integration Study," Vol I and II, ARINC Research Corporation, Anapolis, 1980.

17 Moore, James and Ed Rymit, "SESAME Pre-Processor," US Army Missile Command, 1980. (unpublished)

18 Muckstadt, J. A., "A Model for a Multi-Item, Multi-Echelon, Multi-Indenture Inventory System," Management Science, Vol 20 (1973), p472-481.

19 Muckstadt, John A., "Analysis of a Two-Echelon Inventory System in Which All Locations Follow Continuous Review (S,s) Policies," Cornell University, Ithaca, 1977.

20 Rosenman, B. and D. Hoekstra, "A Management System for High Value Army Aviation Components," Army Inventory Research Office, Philadelphia, 1964.

21 Ross, Sheldon M., "On the Calculation of Asymptotic System Reliability Characteristics," in "Reliability and Fault Tree Analysis," SIAM, 1975, p. 331-350.

22 Sherbrooke, Craig C., "METRIC: A Multi-Echelon Technique for Recoverable Items Control," Operations Research, Vol 16 (1968), p122-141.

23 Stidham, Shaler, Jr., "A Last Word on L = Lambda W," Operations Research, Vol 22, p417-421.

NOTE 1

FAILURES DURING INSTALLATION

Notation

p - probability of failure during installation

t - expected time to failure, during installation, given a failure during installation

MTTR - mean time to repair

TMTTR - expected total mean time to repair including time to get another component if first fails during installation

LDT - mean logistic down time

MTBF - mean time between failure

$r \equiv 1/MTTR$

$f \equiv 1/MTBF$

Case 1: Repair time (given no failure) is deterministic. Failures are exponential.

Then

$$TMTTR = (1-p)MTTR + p\,[t + LDT + TMTTR]$$

$$TMTTR = MTTR + \frac{p[t + LDT]}{1-p}$$

and

$$p = 1 - e^{-(f)(MTTR)}$$

$$t = \frac{1}{p} \int_0^{MTTR} t\, f\, e^{-ft}\, dt$$

$$= \left\{ \frac{1}{f} - e^{-(f)(MTTR)} [MTTR + 1/f] \right\} / p$$

Case 2: Repair Time and Failures are Exponential.

Then

Probability of no installation by time x is e^{-rx}, so

$$p = \int_0^{\infty} f\, e^{-fx}\, (e^{-rx})\, dx$$

$$= \frac{f}{f+r} \int_0^{\infty} (f+r)\, e^{-(f+r)x} = \frac{f}{f+r}$$

$$t = \frac{\int_0^\infty x\, f\, e^{-fx}\, dx}{p} = \frac{1}{p}\,\frac{f}{f+r}\int_0^\infty (x)(f+r)e^{-(f+r)}dx$$

$$= \frac{1}{p}\,\frac{f}{f+r}\,\frac{1}{f+r} = \frac{1}{f+r}$$

When repair times are exponential, the fact that repair was completed before a failure occurred, tells us something about how long the repair took. If

$$\text{MTTRC} \equiv \text{Ex (Repair Time} \mid \text{Repair occurs before failure)}$$

Then, by a derivation analogous to derivation of t,

$$\text{MTTRC} = \frac{\int_0^\infty x\, r\, e^{-fx} e^{-rx} dx}{1-p} = \frac{1}{f+r}$$

This equals t. In other words, something will happen in average time of (f+r). With probability p it is a failure and probability (1-p) it is a repair.

$$\text{TMTTR} = (1-p)(\text{MTTRC}) + p[t + \text{LDT} + \text{TMTTR}]$$

and by algebra

$$\text{TMTTR} = \text{MTTRC} + \left(\frac{p}{1-p}\right)[t + \text{LDT}]$$

$$= \frac{1}{f+r} + \frac{f}{r}\left(\frac{1}{f+r} + \text{LDT}\right)$$

$$= \frac{1}{f+r}\left(1 + \frac{f}{r}\right) + \frac{f}{r}(\text{LDT})$$

$$= \frac{1}{r} + \frac{f}{R}(\text{LDT})$$

NOTE 2

SOLVING MULTIPLICATIVE OBJECTIVE FUNCTION

Notation:

TC : Total Cost, inventory plus backorder

S_i : Total stock (for item i).

UP_i : Unit price

λ : Lagrangian

N : Number of weapon systems supported

B_i : Expected time weighted backorders at user level

SA : Supply Availability

Discussion:

Our objective is:

$$(1) \qquad \text{Minimize: } TC = \sum_i S_i UP_i + (\lambda N)\left[1 - \prod_i \left(1 - \frac{B_i}{N}\right)\right]$$

where the second term on the right hand side expresses expected number of systems down given the Case B approach to operational availability (recall Equation 4.5.7).

Differentiating with respect to S_i

$$(2) \qquad \frac{\partial TC}{\partial S_i} = UP_i + \lambda A_i^- \frac{\partial B_i}{\partial S_i}$$

where

$$(3) \qquad A_i^- = \prod_{j \neq i} \frac{1-B_i}{N} = SA/\left(1 - \frac{B_i}{N}\right)$$

Now suppose we had the optimum solution, S_i^*. Then if we restated the objective as

(4) Minimize: $TC = \sum_i S_i UP_i + \lambda A_T B_i$

and treated A_T as a constant, found from (3) using the SA determined by S_i^*, we would find that the same S_i^* minimized (4) as had minimized (1). This is clear since the derivative of (4) with respect to S_i is (2).

We can rewrite (4) as

(5) Minimize $TC = \sum_i S_i UP_i + \lambda_i B_i$

where

(6) $\lambda_i = \lambda A_T$

Now, we will show later that given an optimum solution of (5),

(7) $$\frac{B_i}{N} < \frac{UP_i}{\lambda_i}$$

Realistically, UP_i/λ_i will not exceed 1. Using this fact and (6), (3) and (7):

(8) $$(\lambda)(SA) < \lambda_i = \frac{(\lambda)(SA)}{1 - \frac{B_i}{N}} < \frac{(\lambda)(SA)}{(1 - UP_i/\lambda_i)}$$

or, by algebra

(9) $(\lambda)(SA) < \lambda_i < (\lambda)(SA) + UP_i$

To summarize, we have found that solving (1) is equivalant to solving

(10) Minirize $TC = \sum_i S_i UP_i + \lambda_i B_i$

$$\lambda' < \lambda_i < \lambda' + UP_i$$

$$\lambda' = (\lambda)(SA)$$

The significance of (10) is that if we minimized (10) for a range of λ', evaluated Supply Availability for each solution by the Case B approach, and drew the availability versus investment curve, our curve and accompanying stockage lists would be the same as minimizing (1).

Comparing (10) to SESAME's objective function, the only difference is that λ should be increased by a maximum of UP_i when item i is being solved. This will normally have very little impact as $UP_i << \lambda$.

Proof that $B_i/N < UP/\lambda_i$

Let U_i be expected stock unavailability, defined here as the probability there is at least one backorder at a random point in time.

Then

$$(11) \qquad U_i \geq \frac{B_i}{N}$$

For example, imagine that whenever there was at least one backorder, there were N. This is a worst case. In this case $U_i = B_i/N$ (e.g. if $U_i = 10\%$, $B_i = (.10)(N)$).

Now in the literature there is a well known result which characterizes U_i when it corresponds to the optimum stockage solution in a single echelon problem with Poisson demand:

$$(12) \qquad U_i < \frac{\text{Cost Per Unit Asset Per Unit Time}}{\text{Cost Per Unit Backorder Per Unit Time}} = \frac{UP_i}{\lambda}$$

The same relationship is directly extended to multi-echelons, because once the optimum solution for echelons 2 thru NECH is fixed, we are left with a classical one echelon problem, at least when using METRIC or TWOPT formulation.

Combining (11) and (12) provides the desired results.

DISTRIBUTION

COPIES	
1	Deputy Under Sec'y of the Army, ATTN: Office of Op Resch
1	Asst Sec'y of the Army (I,L&FM), Pentagon, Wash., DC 20310
	Headquarters, US Army Materiel Development & Readiness Command
1	DRCPA-S
1	DRCMS
1	DRCDMR
1	DRCPS
1	DRCPS-P ATTN: Mr. Boehm
1	DRCMM
1	DRCMM-R
1	DRCMM-RS
1	DRCMM-M
1	DRCMM-MS
1	DRCMM-MP
1	DRCRE
1	DRCIS
1	DRCDE
1	DRCPM
1	Dep Chf of Staff for Logistics, ATTN: DALO-SMS, Pentagon, Wash., DC 20310
1	Dep Chf of Staff for Logistics, ATTN: DALO-SMZ-B, Pentagon, Wash., DC 20310
2	Defense Logistics Studies Info Exchange, DRXMC-D
10	Defense Documentation Center, Cameron Sta., Alexandria, VA 22314
1	Office, Asst Sec'y of Defense, ATTN MRA&L-SR, Pentagon,
	Commander, USA Armament Materiel Readiness Cmd, Rock Island, IL 61201
1	ATTN: DRSAR-MM
1	ATTN: DRSAR-SA
	Commander, USA Communications & Electronics Materiel Readiness Cmd, Ft. Monmouth, NJ 07703
1	ATTN: DRSEL-MM
1	ATTN: DRSEL-SA
	Commander, USA Missile Command, Redstone Arsenal, AL 35809
1	ATTN: DRSMI-S
1	ATTN: DRSMI-D
	Commander, USA Troop Support & Aviation Materiel Readiness Command, St. Louis, MO
1	ATTN: DRSTS-SP
1	ATTN: DRSTS-SPSS
1	ATTN: DRSTS-BA(1)
	Commander, US Army Tank-Automotive Materiel Readiness Command, Warren, MI 48090
1	ATTN: DRSTA-F
1	ATTN: DRSTA-S
1	Commander, US Army Tank-Automotive Research & Development Command, ATTN: DRDTA-V, Warren, MI 48090

COPIES

1 Commander, US Army Armament Research & Development Command, ATTN: DRDAR-SE, Dover, NJ 07801

1 Commander, US Army Aviation Research & Development Command, St. Louis, MO 63166

1 Commander, US Army Communications Research & Development Command, ATTN: DRSEL-SA, Ft. Monmouth, NJ 07703

1 Commander, US Army Electronics Research & Development Command, ATTN: DRDEL-AF, Adelphi, MD 20783

1 Commander, US Army Mobility Equipment Research & Development Cmd, ATTN: DRDME-O, Ft. Belvoir, VA 22060

1 Commander, US Army Natick Research & Development Command, ATTN: DRXNM-O, Natick, MA 01760

1 Commander, US Army Logistics Center, Ft. Lee, VA 23801

1 Commander, US Army Logistics Evaluation Agency, New Cumberland Army Depot, New Cumberland, PA 17070

1 Commander, US Army Depot Systems Command, Chambersburg, PA 17201

1 Commander, US Air Force Logistics Cmd, WPAFB, ATTN: AFLC/XRS, Dayton, Ohio 45433

1 US Navy Fleet Materiel Support Office, Naval Support Depot, Mechanicsburg, PA 17055

1 Mr. James Prichard, Navy Sea Systems Cmd, ATTN: PMS3061, Dept of US Navy, Wash., DC 20362

1 George Washington University, Inst of Management Science & Engr., 707 22nd St., N.W., Wash., DC 20006

1 Naval Postgraduate School, ATTN: Dept of Opns Anal, Monterey, CA 93940

1 Air Force Institute of Technology, ATTN: SLGQ, Head Quantitative Studies Dept., Dayton, OH 43433

1 US Army Military Academy, West Point, NY 10996

1 Librarian, Logistics Mgt Inst., 4701 Sangamore Rd., Wash.,DC 20016

1 University of Florida, ATTN: Dept of Industrial Systems Engr., Gainesville, FL 32601

1 RAND Corp., ATTN: S. M. Drezner, 1700 Main St., Santa Monica, CA 90406

1 US Army Materiel Systems Analysis Activity, ATTN: DRXSY-CL, Aberdeen Proving Ground, MD 21005

1 Commander, US Army Logistics Center, ATTN: Concepts & Doctrine Directorate, Ft. Lee, VA 23801

1 ALOG Magazine, ATTN: Tom Johnson, USALMC, Ft. Lee, VA 23801

1 Commander, USDRC Automated Logistics Mgt Systems Activity, P.O. Box [illegible], St. Louis, MO 63188

1 Director, DARCOM Logistics Systems Support Agency, Letterkenny Army Depot, Chambersburg, PA 17201

1 Commander, Materiel Readiness Support Activity, Lexington, KY 40507

1 Director, Army Management Engineering Training Agency, Rock Island Arsenal, Rock Island, IL 61202

1 Defense Logistics Agency, Cameron Sta, Alexandria, VA 22314

COPIES	
1	Commander, US Army Depot Systems Command, Letterkenny Army Depot, ATTN: DRSDS-LL, Chambersburg, PA 17201
1	Logistics Control Activity, Presidio of San Francisco, CA 94120
1	Operations Research Center, 3115 Etcheverry Hall, University of California, Berkeley, CA 94720
1	Dr. Jack Muckstadt, Dept of Industrial Engineering & Operations Research, Upson Hall, Cornell University, Ithaca, NY 14890
1	Prof Herbert P. Galliher, Dept of Industrial Engineering, University of Michigan, Ann Arbor, MI 48104
1	Mr. Ellwood Hurford, Scientific Advisor, ATCL-SCA, Army Logistics Center, Ft. Lee, VA 23801
1	Prof Robert M. Stark, Dept of Stat & Computer Sciences, University of Delaware, Newark, DE 19711
1	Prof E. Gerald Hurst, Jr., Dept of Decision Science, The Wharton School, University of Penna., Phila., PA 19174
1	Logistics Studies Office, DRXMC-LSO, ALMC, Ft. Lee, VA 23801
1	Procurement Research Office, DRXMC-PRO, ALMC, Ft. Lee, VA 23801
1	Dept of Industrial Engr. & Engr. Management, Stanford University, Stanford, CA 94305
1	Commander, US Army Communications Command, ATTN: Dr. Forrey, CC-LOG-LEO, Ft. Huachuca, AZ 85613
1	Commander, US Army Test & Evaluation Cmd, ATTN: DRSTE-SY, Aberdeen Proving Ground, MD 21005
1	Prof Harvey M. Wagner, Dean, School of Business Adm, University of North Carolina, Chapel Hill, NC 27514
1	Dr. John Voelker, EES Bldg. 11, Argonne National Laboratory, 9700 S. Cass Ave., Argonne, IL 60439
1	DARCOM Intern Training Center, Red River Army Depot, Texarkana, Tx 75501
1	Prof Leroy B. Schwarz, Dept of Management, Purdue University, Krannert Bldg, West Lafayette, Indiana 47907
1	US Army Training & Doctrine Command, Ft. Monroe, VA 23651
1	US General Accounting Office, ATTN: Mr. J. Morris, Rm 5840, 441 G. St., N.W., Wash., DC 20548
1	Operations & Inventory Analysis Office, NAVSUP (Code 04A) Dept of Navy, Wash., DC 20376
1	US Army Research Office, ATTN: Robert Launer, Math. Div., P.O. Box 12211, Research Triangle Park, NC 27709
1	Prof William P. Pierskalla, 3641 Locust Walk CE, Philadelphia, PA 19104
1	US Army Materiel Systems Analysis Activity, ATTN: DRXSY-MP, Aberdeen Proving Ground, MD 21005
1	Air Force Logistics Management Center, ATT: AFLMC/LGY, Gunter Air Force Station, AL 36114
1	Engineer Studies Center, 6500 Brooks Lane, Wash., DC 20315
1	US Army Materiel Systems Analysis Activity, ATTN: Mr. Herbert Cohen, DRXSY-MP, Aberdeen Proving Ground, MD 21105
1	Commandant, ALMC, ATTN: Jon T. Miller, DAS, DRXMC-A, Ft. Lee, VA